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Family of boundary value problems for systems of integro-differential equations and its application

ABSTRACT

of the PhD thesis for the degree of Doctor of Philosophy (PhD)
in the specialty «8D05401-Mathematics»

The relevance of the study. Mathematical modelling of many real processes leads to the study of nonlocal boundary value problems for differential equations. In addition to the theoretical significance of the results of the theory of nonlinear nonlocal boundary value problems for systems of hyperbolic equations, their practical application is found in the study of models of gas dynamics, magnetic hydrodynamics, mathematical biology and other fields.

The study and solution of a family of nonlinear boundary value problems for integro-differential equations is of independent importance, and their application to the identification of properties of solutions to nonlinear nonlocal boundary value problems for systems of hyperbolic equations and the development of constructive methods for their solution emphasises the relevance of the thesis topic.

The purpose of the dissertation is development of methods of research and solution of a family of nonlinear boundary value problems for systems of integro-differential equations, their application to determine the properties of solutions of nonlinear nonlocal boundary value problems for systems of hyperbolic equations and the development of constructive solution methods.

The object of the study is a family of nonlinear boundary value problems for systems of integro-differential equations and its application to a family of nonlinear nonlocal boundary value problems for hyperbolic equations.

The objectives of the research work are:

-To propose modified algorithms of the parameterisation method for finding the solution of linear two-point boundary value problems for a system of linear differential equations, linear two-point boundary value problems for a system of integro-differential equations, to obtain the conditions of their convergence, to prove that these conditions are solvability conditions for the above problems as well.

- To establish and prove necessary and sufficient conditions of the criterion of well-posedness of the family of linear two-point boundary value problems for the system of linear differential equations.

- Determination of the conditions for the continuous dependence of the "isolated" solution of nonlinear boundary value problem for a system of integro-differential equations on small perturbations of the initial data of the problem.

- Show the application of the family of nonlinear boundary value problems for a system of integro-differential equations to the study of a nonlinear nonlocal boundary value problem for one class of hyperbolic equations with mixed derivatives, namely: to show the method of creation of algorithms for finding a solution, to determine sufficient conditions for existence of a solution as a consequence of solvability conditions of a

family of nonlinear boundary value problems for a system of integro-differential equations.

- On the basis of the ideas of the method of parameterisation for finding a solution to a family of nonlinear two-point boundary value problems for systems of linear differential equations, to propose modified algorithms and a method of finding a numerical solution to this problem. Based on these algorithms, develop an algorithm for finding a solution to a nonlinear nonlocal boundary value problem for one class of linear hyperbolic equations.
- Show the implementation of solution finding algorithms in test problems of methods for approximate solution of a nonlinear nonlocal boundary value problem for a system of linear differential equations and for one class of nonlinear hyperbolic equations.

Research methods. To investigate and solve the problems of the dissertation, the parameterisation method of Dzhumabaev, iterative methods for unbounded operator equations and methods of the theory of differential equations were used.

Scientific novelty.

In the dissertation work, the problem on the topic "Family of boundary value problems for systems of integro-differential equations and its application" is considered, a study based on the Dzhumabaev parametrization method is conducted, new results are obtained and presented for defense:

– the parameterization method with modified algorithms for the study and solution of families of nonlinear boundary value problems for systems of differential and integro-differential equations;

– analysis and solution of nonlinear nonlocal boundary value problem for hyperbolic equations with mixed derivatives by investigation and solution of families of nonlinear boundary value problems for systems of integro-differential equations;

– algorithms for finding solutions to the considered problems and sufficient conditions for their convergence;

– sufficient conditions for the existence of a solution to a family of nonlinear boundary value problems for a system of differential equations, a family of nonlinear boundary value problems for a system of integro-differential equations, and a nonlinear nonlocal boundary value problem for one class of hyperbolic equations with mixed derivatives.

Theoretical and practical significance of the results. The results obtained in the dissertation have theoretical and practical significance. The obtained results contribute to the development of the theory of integro-differential equations. The practical significance of the study of the family of nonlinear boundary value problems for systems of integro-differential equations is that its results can serve as a basis for qualitative and quantitative analysis of modelled processes described by families of nonlinear nonlocal boundary value problems for systems of hyperbolic partial differential equations.

Publications. On the subject of the dissertation, 9 papers have been published, including 2 publications in the ranking scientific journals, indexed in the database of Scopus, 2 publications in scientific publications, included in the list recommended by the Committee for Quality Assurance in the Field of Science and Higher Education of the Ministry of Science and Higher Education of the RK for publication of the main

scientific results of scientific activity, 5 publications in the materials of international conferences, including 1 publication in the proceedings of a foreign conference.

– 2 publications in journals included in the list recommended by the Committee for Quality Assurance in the Field of Science and Higher Education of the Ministry of Science and Higher Education of the Republic of Kazakhstan:

1. Temesheva S.M., **Abdimanapova P.B.**, Borisov D.I. On a method for solving a family of nonlinear boundary value problems for ordinary differential equations // Вестник КазНПУ имени Абая, «Физико-математические науки»–2021, –№1(73), -P.70-76.

2. **Abdimanapova P.B.**, Temesheva S.M., Zhumagazikyzy A. On the application of a family of boundary value problems for systems of integro-differential equations // КазНПУ им.Абая, Серия «Физико-математические науки», –2022, –№3(79), -P.7-13.

- 2 articles in scientific journals included in the second and third quartiles (Q2, Q3) according to the Clarivate Analytics Journal Citation Reports and/or have percentages respectively 56, 35 CiteScore in the Scopus database:

1. Temesheva S.M., **Abdimanapova P.B.** On a Solution of a Nonlinear Nonlocal Boundary Value Problem for one Class of Hyperbolic Equation // Lobachevskii journal of mathematics–Kazan Federal University. –2023, –Vol.44(7), –P.2529–2542. (WoS, IF(2022)=0.7, JCI(2022) – Q2; Scopus, 56%)

2. **Abdimanapova P.B.**, Temesheva S.M. Well-posedness criteria for one family of boundary value problems // Bulletin of the Karaganda University. Mathematics series. –2023. –№ 4(112). –P. 5–20. (WoS, IF(2022)=0.6, JCI(2022) - Q3; Scopus, 35%)

The main results of the thesis were reported and discussed at international and foreign scientific conferences and seminars:

1. Temesheva S.M., **Abdimanapova P.B.** On the choice of the initial approximation of the nonlinear nonlocal boundary value problem for the hyperbolic equation // Annual internat. april mathem. conf. in honor of the Day of Science Workers of the Republic of Kazakhstan. – Almaty: Institute of mathematics and mathematical modeling. 5-7 April 2022, P.107-108.

2. **Abdimanapova P.B.**, Temesheva S.M. Initial approximation of the solution of nonlinear nonlocal boundary value problem for one class of systems of hyperbolic equations // IX International Scientific Conference «Problems of Differential Equations, Analysis and Algebra», Aktobe, May 24-28, 2022, P. 101 –103 б.

3. Temesheva S.M., **Abdimanapova P.B.**, Zhumagazikyzy A. On the application of a family of boundary value problems for systems of integro-differential equations // International Scientific Conference "Actual Problems of Mathematics, Mechanics and Informatics" dedicated to the 80th anniversary of Prof. T.G. Mustafin, Karaganda, September 8-9, 2022. P. 146-147.

4. Temesheva S.M., **Abdimanapova P.B.**, Iskakova N.B. On the solvability of the nonlinear nonlocal boundary value problem for system of hyperbolic equations // The international Conference: «Dynamical Systems, Modeling, and Mathematical Sciences», Dubai, September 23-25, 2022, P. 58.

5. **Abdimanapova P.B.** On the solvability criteria of the nonlocal problem for the hyperbolic equation // Annual internat. april mathem. conf. in honor of the Day of Science Workers of the Republic of Kazakhstan. – Almaty: Institute of mathematics and mathematical modeling. 5-7 April 2023, P.60.

Approbation of the work. The main results were reported and discussed at the following scientific events:

1. The traditional International Scientific April Conference. Institute of mathematics and mathematical modeling. Almaty, (5-8 April 2022);
2. IX International Scientific Conference " Problems of Differential Equations, Analysis and Algebra ". K. Zhubanov Aktobe Regional University, Aktobe (May 24-28, 2022).
3. The international Conference: «Dynamical Systems, Modeling, and Mathematical Sciences», Dubai (September 23-25, 2022)
4. International Scientific Conference "Actual Problems of Mathematics, Mechanics and Informatics" dedicated to the 80th anniversary of Prof. T.G. Mustafin. E.A. Boketov Karaganda University, Karaganda (September 8-9, 2022).
5. The traditional International Scientific April Conference. Institute of mathematics and mathematical modeling. Almaty, (5-7 April 2023);
6. Scientific seminar of the Department of Differential Equations. Al-Farabi Kazakh National University. Almaty, (May, December 2021г., May, December 2022г., May, 2023г.);

The structure and scope of the thesis. The dissertation consists of an introduction, three sections (each section includes 3 subsections), a conclusion, and a list of 118 used sources.

The introduction includes a general description of the work, the relevance of the research topic, the main goals and objectives of the research, the object and method of research, scientific novelty and the main results presented for defence, the theoretical and practical significance of the research, approbation of the work, publications, the scope and structure of the thesis and a summary of the work.

In the first section, linear two-point boundary value problems for a system of ordinary differential equations are considered. Algorithms for finding approximate solutions of the problem under study on the basis of the parameterisation method of Dzhumabaev are proposed and sufficient conditions for their fulfilment and convergence are obtained. Sufficient conditions for the existence of a solution to a family of linear two-point boundary value problems for a system of ordinary differential equations are determined, and necessary and sufficient conditions for correct solvability are established.

In the second section, a boundary value problem for a family of linear differential equations obeying a family of nonlinear two-point boundary conditions is considered, that is, a family of nonlinear two-point boundary value problems for systems of linear differential equations is investigated. The conditions of existence of an isolated solution of the considered problem are determined. An algorithm of numerical method of finding the solution of a family of nonlinear boundary value problems is proposed. Examples of the proposed method of finding a numerical solution to the problem are considered: in the first one the matrix of the differential

equation depends only on the parameter of the family and the fundamental matrix is explicitly written; in the second example, the matrix of the differential equation is variable, i.e. the values of the fundamental matrix are determined at each value of the independent variable; in the third example the problem is considered whose solution is unknown. These examples show that the parameterisation method gives good results when certain conditions are met. The study and solution of nonlinear nonlocal boundary value problems for hyperbolic equations with mixed derivatives are proposed to be investigated and solved by reducing them to a family of nonlinear boundary value problems for differential equations.

In the third section, we consider a nonlinear boundary value problem for a family of integro-differential equations and its applications. An algorithm for finding a solution to the problem is proposed. The conditions of fulfilment and convergence of the proposed algorithms are obtained, i.e. sufficient conditions for the existence of an isolated solution in some set of the family of nonlinear boundary value problems for integro-differential equations are determined. The concept of "isolated" solution is introduced. The continuous dependence of the "isolated" solution on small perturbations of the initial data of the problem is established and the conclusion is justified. A nonlinear nonlocal boundary value problem for a system of hyperbolic equations is considered. The study and solution of nonlinear nonlocal boundary value problem for hyperbolic equations with mixed derivatives is carried out by studying and solving families of nonlinear boundary value problems for systems of integro-differential equations. An example defining the results of the research by numerical solution method is presented to demonstrate the application of nonlinear boundary value problems for systems of integro-differential equations.

The final section presents the conclusions and the main results of the research performed in the dissertation.